Lecture Notes (YONSEI UNIVERSITY)

Lecture 00

Introduction to the class

Hong-Yeop Song

School of Electrical and Electronic Engineering

hysong@Yonsei.ac.kr

Course Basic

EEE3315-01 (EE Linear Algebra)

Room	real-time zoom + recorded video	Time/Place	**1 hour - Tue 5, D603** 2 hours - recorded lecture (upload on Thursday 9am)
Professor	Song, Hong-Yeop	URL	http://coding.yonsei.ac.kr/~hysong
Office	C422 (Third Engineering Building)	Cell Phone	010-7661-4861 (urgent only)
Email, office-hour	hysong@yonsei.ac.kr Kakaotalk: hysong2013 (ALWAYS) office-hour : 1 hour after class is reserved		

Send me a message via KAKAOTALK (hysong2013) including

your name,
student id number,
email,
handphone number,
any comment including your motivation of taking this class, etc.

Due this weekend!!!

- Textbook 1: Linear Algebra and its applications (G. Strang) 4th edition
- Textbook 2: Linear Algebra Done Wrong (S. Treil)
- Textbook 3: Linear Algebra (Georgi E Shilov)

PDF files can be found on GOOGLE SEARCH.
I will also distribute thru Learnus or KAKAOTALKROOM

선형대수의 역사는 길다. 1750년의 연립1차방정식을 풀기 위한 Kramer의 규칙이 모체가 되고 1800년경의 Gauss 소거법을 거쳐 1850년 무렵에 Caley와 Sylvester에 의해 행렬이론이 세워진 후, 20세기 들어 대학 수학과정의 기본과목으로 정립되었다고 볼 수 있다.

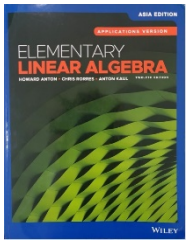
가장 단순한 선형(1차)시스템을 다루는 대수학이라는 특성 때문에 선형대수는 자연스럽게 추상대수나 함수해석학과 같은 고등수학의 출발점이 된다. **또한 선형시스템이 거의 모든 과학, 공학의 모델링에서 출현하기 때문에 선형대수는 과학, 공학에 직접 응용된다**. 더욱이 선형대수의 주인공들인 행렬이나 선형변환은 컴퓨터를 이용한 각종 수치계산 과 응용이론 개발에 직결되는 특성이 있기 때문에 현대적 조류에 부합된다.

이런 이유로 오늘날 선형대수는 수학 전공자 뿐만 아니라 이공계의 모든 전공, 더 나아가서는 경영, 경제, 사회과학의 진지한 분석자들의 필수 교과로 점점 인기를 더해 가고 있다.

(후략)

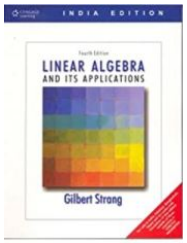
- 어느 번역서 서문 중에서 발췌

Top 10 references for Linear Algebra



1. "Elementary Linear Algebra with Applications" by H Anton

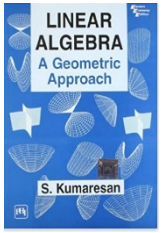
"Elementary Linear Algebra with Applications" Book Review: Elementary Linear Algebra, gives an elementary treatment of linear algebra that is suitable for students in their freshman or sophomore year. The purpose is to present the fundamentals of linear algebra in the clearest possible way. A simple explanation is the main consideration. Calculus is a prerequisite along with there are clearly labeled exercises and examples for students who are studying calculus. Those exercises can help you strengthen your skills. There is no need for technology, if someone would like to use calculators with linear algebra capabilities, exercises have been included at the ends of the chapters that allow for further exploration of chapter's contents.



2. "Linear Algebra and its Applications" by G Strang (This is the TEXTBOOK)

"Linear Algebra and its Applications" Book Review: This book is written in a student-friendly style and teaches real Mathematics. The student will experience a gear shift while going through it especially from the Introduction of Vector Spaces. Throughout the book, the theory is well presented, allowing students to have a deep dive from pure mathematics to applied mathematics. The exercises in the book are well designed and have many new problems added to it along with **Singular Value Decomposition**. Here a student can learn how the algorithm fits linear Algebra with an introduction to complex numbers. This book is a bonus for Computer Science students.

3. "Linear Algebra – A Geometric Approach" by S Kumaresan

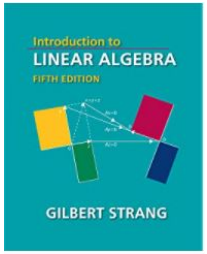


"Linear Algebra – A Geometric Approach" Book Review: This book is highly readable, clear, and concise on Linear Algebra and is for undergraduate courses in mathematics. The key focus is on analytical geometry explanations to make a clear view of the topic. From the beginning, the topic is presented as simultaneous linear equations and their geometric interpretation which is the theme of the subject. Abstract algebraic concepts with geometric notions are a distinguishing feature of this book. It is designed such as to help students in Multivariable calculus and differential geometry. The explanation and concepts are well presented here.

4. "Advanced Engineering Mathematics" by E Kreyszig (공업수학)

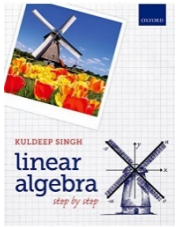
"Advanced Engineering Mathematics" Book Review: This book is well designed to let you taste the real cream of engineering mathematics and is also popular among Engineering students. This book is also a textbook of mathematics in various Engineering colleges as the problems in the books are so well designed that it will let you revise all your basic concepts in mathematics. This book is thoroughly updated and is streamlined to reflect the upliftment in the field. It covers all the advanced topics of engineering mathematics like Linear Algebra, Vector Calculus, Partial Differential Equation, Optimisation, Graphs, Complex Analysis, Statics & Probability so that students can relate to practical problems and can grasp their best.

5. "Introduction to Linear Algebra" by Gilbert Strang



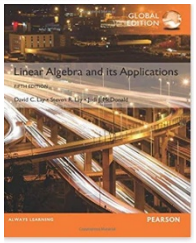
"Introduction to Linear Algebra" Book Review: This book is well described and simple to Linear Algebra and has many breakthroughs within the subjects itself. This is an easy to read textbook that incorporates all the topics essential for detailed knowledge of the subject .The book teaches theory behind everything so it's much engaging with the subject. The textbook includes many challenging problems for better grip on the topic. The book has been divided into seven different topics such as differential calculus, graph theory, statics, Fourier transformation methods, LPP and computer graphics and is complete for a deep overview of the subject.

Top 10 references for Linear Algebra



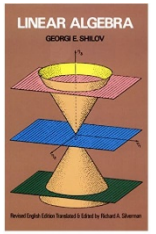
6. "Linear Algebra: Step by Step" by Kuldeep Singh

"Linear Algebra: Step by Step" Book Review: This book has a large number of examples and their step-by-step explanation for each topic. The topic is simplified in a way that allows distance learning with a concise solution to the set of problems freely available. This book consists of miscellaneous exercises at the end of each chapter which covers questions from past papers and also from various university exams helping the reader to boost his or her confidence. It also has short historical biographies of the leading players of Linear Algebra. This book is dynamic and has engaging style which includes question answer tests for the reader to make a clear view of the methods rather than rote learning. It also has exclusive interviews with the professionals who use the topic in their real life.



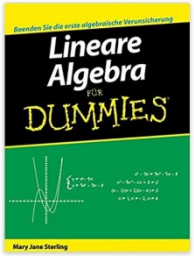
7. "Linear Algebra and Its Applications" by David C Lay and Steven R Lay

"Linear Algebra and Its Applications" Book Review: In this book, the traditional Linear Algebra texts are relatively easy to understand for students as the topic here is presented here in a familiar manner and with a concrete setting. Students often face problems with abstract topics like linear independence, spanning, subspace, vector space, transformations, and the book takes good care of this as the topic needs time to assimilate. These topics are important and understanding the topics is vital and this book makes it more accessible by its simple and explanatory demonstration which is easy to understand for students.



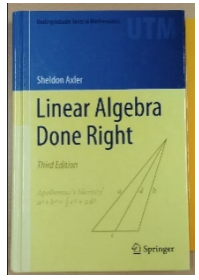
8. "Linear Algebra" by Georgi E Shilov

"Linear Algebra" Book Review: In this book, the course has been made more accessible and useful. This book goes from the elementary level and easily catches up with advanced topics along with covering all the standard topics of an undergraduate or a beginning graduate course. The material here is demonstrated here in a simple style. Here the problems are included and their answers at the back. This book is good for students who want to learn techniques as it has an abundance of problems and examples so as to give the reader a good experience of applicability if the topic and is also useful for self-study and the classroom as well.



9. "Linear Algebra For Dummies" by Mary Jane Sterling

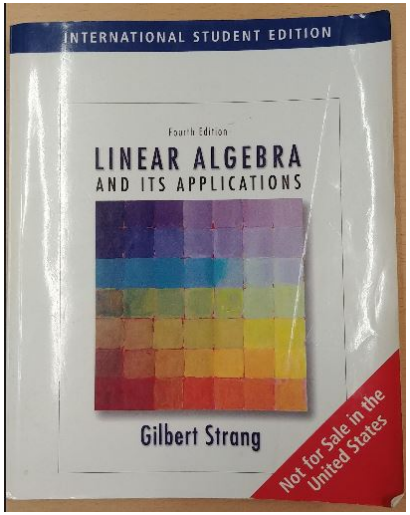
"Linear Algebra For Dummies" Book Review: This book is an easy to follow guide for the topic which offers a clear view of real-world application of Linear Algebra in many areas like programming, Engineering. This book is near a college-level linear algebra course, In which students study the basic operations and then abstract topics like vector space, linear transformation, determinants, and eigenvalues & vectors. This book gives students both theoretical and practical ways to solve different types of problems. The book presents the information in such a way that the reader can not only know how to solve it but also why to solve it.

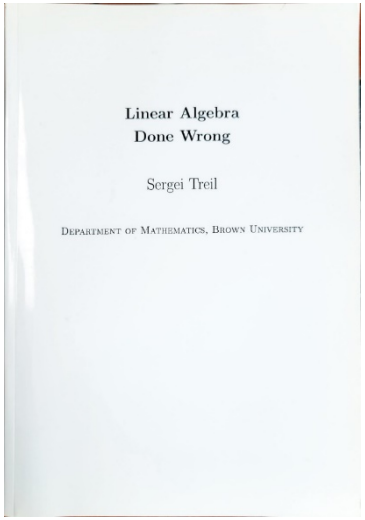


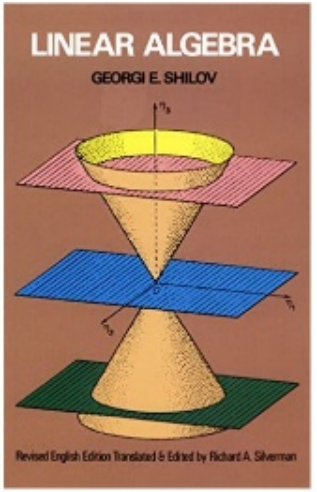
10. "Linear Algebra Done Right" by Sheldon Axler

"Linear Algebra Done Right" Book Review: This book for linear algebra is aimed at both graduate and undergraduate students. The narrative approach of this book to the topic focuses on the central goal of the topic so that students can understand the linear operations on finite-dimensional vector spaces. The book then deals with topics like linear maps, eigenvalues, and eigenvectors. Inner-product spaces, leading to the finite-dimensional spectral theorem and its consequences. In this book the concepts are concise and proofs are simple with a variety of interesting exercises on each chapter to help students in understanding the topic well and manipulate the objects of linear algebra. This book comes with many updated examples to illustrate the key ideas of linear algebra which include product spaces, quotient spaces, and dual spaces. There is no prerequisite for this book except mathematical maturity.

Textbooks







all three PDF files will be distributed in class

Introducing Hong-Yeop Song

- Education
  - Ph.D. University of Southern California, Los Angeles, CA, Dec 1991.
  - MSEE University of Southern California, Los Angeles, CA, May 1986.
  - BSEE Yonsei University, Seoul, Korea, Feb 1984.
- Employment History
  - Professor, Yonsei University, Seoul, Korea, Sept 1995 – Current.
  - Visiting Professor, University of Waterloo, Canada, March 2002 – Feb 2003.
  - Senior Engineer, Qualcomm Inc., San Diego, CA, April 1994 – Aug 1995.
  - Research Associate, Communication Sciences Institute, USC, Los Angeles, Jan 1992 – March 1994.
- Research Interest in General
  - Pseudo-random Sequences and related Mathematics with some applications
  - Sequence Designs for Digital Communications and Cryptography
  - Error Correcting Codes for Reliable Communication and Storage
  - Wireless Communications/Mobile Communications/Satellite Communications
- Research Lab: Channel Coding Lab (http://coding.yonsei.ac.kr)
  - 5 PHD-program students
  - 2 MS-program students
- Graduate Students and Post-Docs Supervising
  - Supervised 50 MS and 11 PHD Thesis's
  - Supervised 3 post-docs

담당교수 소개

- 84년 2월: 연세대학교 전자공학과 졸업
- 88년 2월: 육군3사관학교 소위 임관 및 제대 - 병역필
- 91년 12월: University of Southern California 공학박사
- 92년~93년: USC Post-doc 연구원
- 94년~95년: Qualcomm San Diego 수석연구원 (CDMA이동통신 표준팀 근무)
- 95년 9월부터 현재: 연세대학교 교수

- 전공분야
  - **디지털통신**분야: 무선통신, 이동통신, CDMA, PN codes(sequences), Error-correcting codes
  - **수학**분야: 대수학, 이산수학, 정수론, Combinatorial Designs
- 학회활동
  - **IEEE Information Theory Society**, IEEE Communications Society
  - **한국통신학회**, **한국 정보보호학회**, 대한전자공학회
  - **Mathematical Association of America**
  - **대한수학회**, 한국수학사학회
- 교과목
  - (학부) 데이터구조, **확률과 랜덤변수**, 디지털통신, 전기전자공학수학, **전기전자선형대수**
  - (대학원) **오류정정부호**, 고급선형대수, **통신신호설계**, 정보이론, 암호이론, 등등
- **채널코딩 연구실** http://coding.yonsei.ac.kr 참조

With undergrad students

1995: 연세대 부임

2002: 분반학생들과 강촌MT

2014: 3월 하늘공원 길
2012년도 Freshmen Seminar 수강생들과

학회상
수상자
소개

2017년도 대한수학회상 공로상 송홍엽 회원 수상

송홍엽 회원
(연세대학교)

송홍엽 회원은 서던캘리포니아대학교(Univ. of Southern California) 전기공학과에서 디지털 통신 이론을 전공하고 서던캘리포니아대학교에서 박사후 연구원 2년, 샌디에이고에 위치한 퀄컴사(Qualcomm Inc.)에서 수석연구원(Senior Engineer)으로 약 2년간 근무한 후 1995년 연세대학교 전자공학과 교수로 부임하였다.

송홍엽 회원의 박사학위 논문 주제는 마방진, 직교라틴방진 등과 관련이 있는 Tuscan 방진과 그 응용에 관한 내용이었다고 한다. Tuscan 방진은 라틴방진의 특수한 형태로서 (1) 모든 행은 n개의 서로 다른 심벌을 포함하며 (2) 배열 전체에서 가로로 인접한 심벌의 순서쌍이 모두 달라야 한다. 이러한 배열의 존재와 구성(construction) 그리고 응용에 대한 송홍엽 회원의 관심은 디지털 통신시스템의 신호 설계 및 채널 코딩 설계와의 밀접한 관련성 때문이다. 결국 이러한 시스템의 성능을 결정하는 가장 치명적인 성능지표를 최적화해두고서 나머지 가능성을 바라보는 접근 방식인 셈이다.

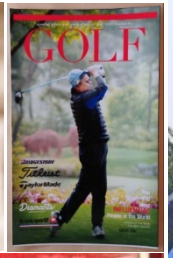
한편, '직교라틴방진'이라는 개념은 1776년에 레온하르트 오일러(Leonhard Euler)에 의해서 제기되었는데 이는 놀랍게도 마방진을 구성하기 위한 도구로 제안되었고, 오늘날 조합론의 효시로 인정받고 있다. 1997년에 송홍엽 회원은 2001년에 작고한 헝가리의 라틴방진 전문가 요제프 데니스(József Dénes) 박사로부터 9차 직교라틴방진의 기록이 조선시대 최석정(1646~1715)의 저서 《구수략》에 기록되어 있다는 사실을 처음 듣게 되었다. 이 사실은 한상근 교수(KAIST)가 1993년에 한국수학교육학회지에 발표하여 세상에 알려지게 되었는데, 이를 한상근 교수가 슬로바키아의 마틴 바차(Martin Bača) 교수에게 전하고, 그가 다시 요제프 데니스 박사에게 전한 내용이 송홍엽 회원에게 전달된 것이다. 이때 송홍엽 회원은 이 분야의 전문가의 일원으로서 커다란 숙제가 주어진 느낌이었다고 한다. 왜냐하면 《구수략》은 오일러의 발표보다 최소 60여 년 앞서기 때문이다. 이후 그간 개인적으로 유대 관계를 유지했던 조합론 분야, 특히 조합론 디자인 분야의 미국수학회 멤버들과 관련 소식을 주고받으며, 결국 최석정과 《구수략》이 2007년 《조합론 디자인 편람(Handbook of Combinatorial Designs Second edition, Chapman & Hall / CRC 출판)》에 언급되도록 만들었다. 라틴방진에 대한 개념은 훨씬 더 오래 전이지만 직교라틴방진의 개념은 최석정의 기록이 사실상 최초이기 때문에 더욱 의미 있는 결실이었다. 그 결과 2013년 대한민국 과학기술 명예의 전당에 수학계 인사로는 두 번째로 최석정이 헌정되기에 이르렀다.

송홍엽 회원은 최석정의 9차 직교라틴방진에 대한 내용과 이에 대한 많은 연구가 이어져서, 그 결과가 국내뿐만 아니라 국제 학회에도 발표되고, 직교라틴방진이나 오일러방진이 아닌 '직교최석정방진'이라고 전 세계에서 불리는 날이 오기를 바래본다고 한다. KMS

2017 Special Contribution Award from Korean Mathematical Society

2017년 대한수학회 공로상 수상
(대한수학회장 이향숙 교수)

Personal



Why do we have to study **Linear Algebra**

- **Fundamental theory of** ALMOST EVERY field of sciences and engineering
- **Communications**, **signal processing**, **network analysis**, **controls**, **computer sciences** including **AI, and a lot more**
  - Every single non-linear equation are modelled as simultaneous linear equations
  - Every real-coefficient higher order differential equation is modelled as simultaneous linear differential equations
  - Every non-linear processing is modelled or approximated as a linear processing
  - Every linear processing and simultaneous linear equations can be represented by some MATRIX
- This class will train **your attitude of mathematical thinking** and **proof techniques** which is essential in your thesis writing.
  - Try to work on all the HW problems ON TIME.
  - READ the text as much as possible. We have two+1 textbooks.

우리는 왜 선형대수학을 공부해야 하는가?

- 통신/신호처리/네트워크/제어 그리고 컴퓨터공학 전분야(AI/DATA분석)에서 필수 도구로 사용함. 위의 전공과목을 잘 생각해보면 거의 대부분이 선형대수학의 기본 이론을 사용하여 진행됨. 예를 들자면,
  - 모든 **비선형 방정식은** 선형화를 거쳐서 "**선형 연립방정식**"의 형태로 모델링
  - 모든 **실계수 미분방정식은** "**선형 연립방정식**"으로 모델링
  - 모든 (통신/압축/제어/랜덤) **신호처리 과정은** "**선형변환**"으로 모델링
  - 모든 **선형 변환과 선형 연립방정식**은 **"행렬"로 표현가능하면 이를 "대각화"**하여 처리
  - **인공지능의** Deep learning 혹은 **데이터분석** 분야에서도 매우 큰 규모의 **선형 연립방정식**이 매우 중요.
- 수학적 사고방식 (증명 및 구조체 분석) 배우기에 매우 적합한 초보적인 내용이며, 이러한 내용은 위의 전공분야 연구활동 (논문 읽기, 논문 쓰기)에 필수적이기 때문에, 대학원 진학(해외 유학)을 계획한다면 필수적으로 수강해야 함.
  - 예습/복습(HW문제풀이)는 물론이고, 교재와 참고문헌 적어도 한 권씩 동시에 공부하기를 추천함

연세대학교 2021학년도 2학기 주관식결과

기관	학부	학기	2021학년도 2학기	담당교수	송홍엽
학점	3	학정번호	EEE3315-01-00	교과목명	전기전자선형대수

주관식 문제	1. Satisfaction

No
다양한 수학 주제들로 구성해 매우 유익했습니다. 수업 자료 잘 정리되어 있어서 복습하기 용이했고, 무엇이 중요한지 알기 편했습니다. Toy example이 이미 적혀 있어 스스로 다 계산해 볼 필요 없이 바로 읽기만 해도 이해된다는 것이 정말 좋았습니다. (Text Book 읽어볼 시간이 부족했는데 강의 자료에 모든 것이 있는 것 같습니다.) 대학 와서 들은 강의 중 최고였습니다 .
The class materials were useful and the content of the class was very valuable for our major.
.
Too much load and lecture contents, though project is good.
To many assignment, and not enough explanation with lecture
교수님이 학문에 열정적이시나, 과제가 많고 수업자료가 불친절하다. 대폭 개선했음
so hard
전에도 선형대수 수업을 들었었지만, 선형대수의 진면목을 알지 못했었다. 이번 수업은 선형대수라는 과목을 왜 들어야 하는지 들으면서 납득이 되는 수업이었다. 행렬에 내재된 법칙들과 이를 활용해 원하는 값을 끌어내는 공학적 방법론에 대해 알게 되어 매우 좋았다.
This class covered many subjects

연세대학교 2021학년도 2학기 주관식결과

기관	학부	학기	2021학년도 2학기	담당교수	송홍엽
학점	3	학정번호	EEE3315-01-00	교과목명	전기전자선형대수

주관식 문제	3. Intellectual Challenge

꼼꼼한 내용과 적절한 양의 선정된 과제로 선형대수학으로 수학을 입문하는데 있어, 혹은 선형대수를 이미 알더라도 이 수업을 들어야 한다고 추천하고 싶습니다. 공리와 Theorem들이 밑바닥부터 차근차근 연결되어 이해되고 체계화되었고 수학지식 빈 구멍들이 많이 매워졌습니다. 앞으로 수학을 어떻게 공부해야 할지도 알게 되었습니다.

강의 자료로만 보면 지금까지 대학에서 들었던 어떤 수업보다도 유익했던 것 같다. 다만 그 난이도와 진도를 나가시는 속도, 모국어가 아닌 영어로 전달 받는 데에서 오는 이해의 저하가 수업을 들을 때 마다 매우 아쉬웠다. zoom 수업으로 진행할 때 중간중간 한국어로 번역해서 설명해주실 때가 있었는데, 그 때 전달력이 훨씬 좋으셔서 영어가 아닌 한국어로 전달 받았다면 어땠을까 하는 아쉬움이 있어 Agree를 택했다.

Course Contents

week #	lec #	Topic	Details	HW/Project	
1 9.1(목)-9.7(수)	1	Introduction to the class Chapter 1 Matrices and Gauss Elimination	▪ Introduction to the class ▪ Simultaneous Linear Equations - row/column view ▪ Gauss Elimination and LDU decomposition	▪ HW#01	▪ (9.5. - 9.7.) 수강신청 확인 및 변경
2	2	Extra Topic of Important Concepts	▪ Modular arithematic and binary field	▪ HW#02	▪ (9.9. - 9.12.) 추석연휴 및 대체공휴일
3	3	Chapter 2 Vector Spaces – part 1	▪ (scalar) fields ▪ Vector spaces over a scalar field ▪ Four fundamental subspaces of a matrix	▪ HW#03	
4	4	Chapter 2 Vector Spaces – part 2	▪ Introduction to Linear Transformations ▪ Linear Transformations and Matrices	▪ HW#04	▪ 9.20-22 추석연휴
5	5	Chapter 4 Permutations and Determinant	▪ Permutations ▪ Determinants	▪ HW#05	
6	6	Extra Topic of Important Concepts	▪ Dual Space and Expansion Formula ▪ Quotient Space and First Isomorphism Theorem		
7	-	Summary of first Half Semester	10.13 Thu meeting will be at late night with zoom We will have QNA session for all the previous contents		
8		**midterm exam (10.18 classroom 1 hour)**	▪ midterm exam ▪ 10.20 Recorded video will discuss EXAM SOLUTION		(10.20. - 10.26.) 중간시험
9	7	Chapter 3 Orthogonality – part 1	▪ Orthogonal Vectors, Inner Product Space and Subspaces ▪ Cosines and Projections	▪ HW#06	▪ (10.27. - 10.31.) 수강철회
10	8	Chapter 3 Orthogonality – part 2	▪ Projection and Least Square ▪ Orthonormal Basis and Gram-Schmidt Process, QR decomposition	▪ HW#07	
11	9	Chapter 5 Eigenvalues and Eigenvectors–part 1	▪ Introduction to eigenvectors and eigenvalues ▪ Linear difference equations, Markov Matrices and Spectral Theorem ▪ matrix exponentials and linear differential equations	▪ HW#08	
12	10	Chapter 5 Eigenvalues and Eigenvectors–part 2	▪ Complex matrices ▪ Similar transformation ▪ Schur's Lemma on Triangulization and Normal Matrices	▪ HW#09	
13	11	Chapter 6 Positive Definiteness and SVD - part 1	▪ Positive definite matrices and Cholesky Decomposition ▪ Congruence transformation	▪ HW#10	
14	12	Chapter 6 Positive Definiteness and SVD - part 2	▪ Singular Value Decomposition	▪ Project	
15	-	Summary of Second Half Semester			(12.8. - 12.14.) 자율학습
16		**final exam (2 hour)**			(12.15. - 12.21.) 기말시험

Course operation

- Every Thursday morning
  - ✓ Recorded Video is uploaded to Learnus (at about 9 am on Thu or Wed midnight)
  - ✓ You have to watch these video and work on HW problems (until Tues class meeting)
  - ✓ Prepare to submit your HW problem Solving Report (Handwritten) by Tuesady classroom meeting

- Every Tuesday 5 (1:00 pm - 1:50 pm)
  - ✓ This is classroom meeting at D603
  - ✓ I will explain some selected problems on HW problems set
  - ✓ I will discuss on any subject (from your questions) in the recorded video uploaded last week.
    - ✓ PLEASE make sure that you propare for a question or two for the classroom discussion
  - ✓ I will reserve Tuesday 6 (2-3 PM) for your personal questions on the subject at my room after class

- TA and I will be available for your questions after you submit the report after Tuesday class
  - ✓ Try to talk thru KAKAOTALK classroom.
  - ✓ You may visit TA on some specified time of the week. (TBD and TBA)
  - ✓ You may come to me on Tue6. Otherwise, send me message thru KAKAOTALK

- You have to check the electronic tag for attendance.
  - ✓ Dont be late on Tue class meeting. 5 min or more late will be counted as an absence.
  - ✓ For all types of absence, please submit an official document for your absence to me personally thru email at hysong@Yonsei.ac.kr
  - ✓ Please try not to be absent on Tue classroom meeting. Come to the class will be critical for your study.

Preparation of your HW report

- Use **A4 size paper** and only one-side.
  - Use A4 size for Notepad writing
- Put your **name and Student id number** at the top of the first page.
- **Write down** in your own handwriting the following:
  - I claim that this work was done only by myself without help from any others. and SIGN, and put DATE
- Use **handwriting ALWAYS** unless otherwise specified.
- 50% penalty if your report does not have the above form.

- 마감은 매주 화요일 수업시간 시작직전.
- 마감이후 받지 않습니다. 마감을 꼭 지키세요.

- Due time is **the beginning of the class in Tue 6.**
- No later submission will be accepted.

Evaluation Mechanics

- Weekly HW total : 200 points = 20 points x 10 times
- Midterm/final exams : 350 points (=150+200)
- Project report : 50 points
- TOTAL 600 points.

You will get A for more than 420 (70%) and

get B for more than 330 (55 %)

Some Preliminary Concepts on "tuples"

What do we study in Linear Algebra

(1) **tuples** (and their **equations**) --- SIGNALS (and RELATIONS)

(2) **linear functions on tuples** --- SYSTEMS (Linear Systems)

Following a few pages will be a preview of what you will study in this class, and they contain some important concepts.

I hope that it is a good review for you.

We will study (1) **tuples** and **equations**

- We now discuss and study **equations** for "**tuples**" instead of **scalars** or **numbers**
  - $3x = b$ is an **equation** for the unknown number x with coefficient number 3 and constant number b
  - $\begin{bmatrix}2 & 1\\ 3 & 0\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}a\\ b\end{bmatrix}$ is an **equation** for the unknown **tuple** $\begin{bmatrix}x\\ y\end{bmatrix}$ with coefficient matrix $\begin{bmatrix}2 & 1\\ 3 & 0\end{bmatrix}$ and constant tuple $\begin{bmatrix}a\\ b\end{bmatrix}$
    - Homogeneous equation when $\begin{bmatrix}a\\ b\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$
    - Inhomogeneous or non-homogeneous equation when $\begin{bmatrix}a\\ b\end{bmatrix} \neq \begin{bmatrix}0\\ 0\end{bmatrix}$

- Discuss and study new rules for **addition/multiplication of these tuples** and also with numbers
  - How to add tuples? $\begin{bmatrix}x\\ y\end{bmatrix} + \begin{bmatrix}w\\ z\end{bmatrix} = ?$ or $\begin{bmatrix}x\\ y\end{bmatrix} + \begin{bmatrix}a\\ b\\ c\end{bmatrix} = ?$
  - How to multiply tuples? $\begin{bmatrix}x\\ y\end{bmatrix} \times \begin{bmatrix}w\\ z\end{bmatrix} = ?$ or $\begin{bmatrix}x\\ y\end{bmatrix} \times \begin{bmatrix}a\\ b\\ c\end{bmatrix} = ?$ or $\begin{bmatrix}x & y\end{bmatrix} \times \begin{bmatrix}w\\ z\end{bmatrix} = ?$ or $\begin{bmatrix}w\\ z\end{bmatrix} \times \begin{bmatrix}x & y\end{bmatrix} = ?$
  - Can we add a number to a tuple? $2 + \begin{bmatrix}x\\ y\end{bmatrix} = ?$
  - Can we multiply a number to a tuple? $2 \times \begin{bmatrix}x\\ y\end{bmatrix} = ?$

We will study (1) **tuples** and **equations**

NAME and NOTATION:

- We use the name "**n-tuple**" for an array $[a_1\ a_2\ a_3\ \cdots\ a_n]$ or $\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{bmatrix}$

instead of "**vector** of length n" or "n-dimensional **vector**"

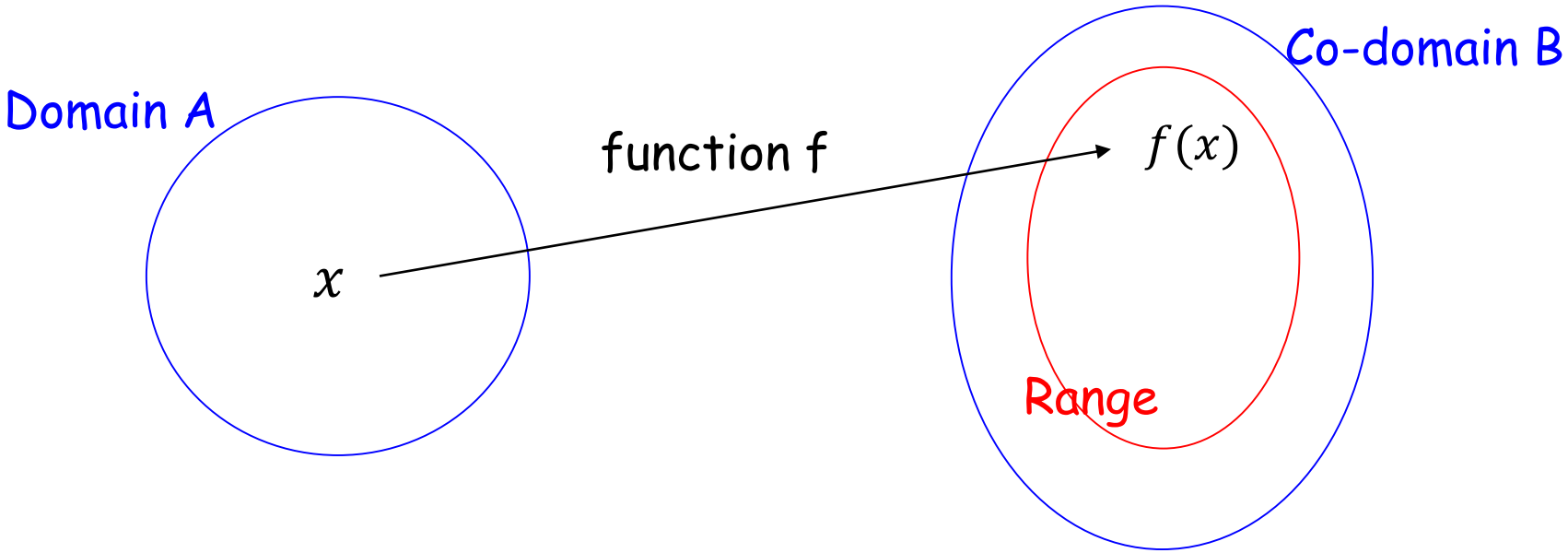
The pre-concept of **magnitude and direction** could be wrong.
Sometimes, it does not have a magnitude.
Sometimes, it is hard to determine its direction.

The pre-concept of **dimension** could be mis-leading.
The number of components is not at all related to "DIMENSION."
For example, a 3-tuple can be a member of 2-dimensional vector space.

- Sometimes, we use both for convenience.

Function is a mapping from a set

- Domain is a set on which a function is defined
- Co-domain is a set to which a function value belongs
- Range is a subset of Co-domain in which every member must be an image of the function.



- A **graph** of a function $f(x)$ is the set of points $(x, y = f(x))$ in $x - y$ plane.

1-1 and/or onto

- Given sets A and B, a mapping f from A to B is a function if for EVERY a in A, the corresponding value $f(a)$ is well-defined, and it belongs to B.

- The range(f) is defined as $\{ f(a) \mid a \in A \}$, which is a subset of B.
- If the range(f)=B, then the function is said to be from A **onto** B.
- In general, any function is from A **onto** the range(f).
- To show that a function f is from A **onto** B, you must show that **every** b in B is an image under f of some element a in A.

- **For any** b in the range(f), there exists **at least one** a in A such that $f(a)=b$.
- If there exists **exactly one** a in A with $f(a)=b$ **for any** b in B, then the function is said to be **1-to-1** (or, **1-1**).
- To show that a function f is 1-1 from A to B, you must show that $f(a1)=f(a2)$ implies a1=a2 **for any** a1 and a2 in A.

We will study (2) linear functions on tuples

- We NOW consider the functions with **the domain** in which the **addition** is defined
- For example,
  - real numbers, complex numbers, You must be familiar with the addition here
  - the binary numbers {0,1} with mod 2 operations 0+0=0, 1+0=0+1=1, 1+1=0
  - the set of 2-tuples $\begin{bmatrix} a \\ b \end{bmatrix}$ over the reals
  - the set of 2-tuples $\begin{bmatrix} a \\ b \end{bmatrix}$ over the binary numbers {0,1} with mod 2 operation

$$\begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} a+c \\ b+d \end{bmatrix}$$

addition of tuples

addition of reals or binary numbers

- What is a **linear function**?
  - **A linear function preserves the addition from domain to range.**

- $f(x) = 5x$ is a linear function from R to R
  - $f(a+b) = f(a) + f(b)$ for any real a and b.

addition in domain

addition in range

EITHER
send the sum a+b to the range by f
OR
send a and b individually to the range by f and then add the results.
Both results are the same.

  - Be careful of 4 different additions in this page!
- $g(x) = 3x + 1$ is an **affine** function from R to R **(not linear)**

We will study (2) linear functions on tuples

- A function that sends every 2-tuple $\begin{bmatrix} x \\ y \end{bmatrix}$ to the number $x + 2y$ is a **linear function** from R^2 to R
  - We may write $f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = x + 2y$
  - We may write $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = x + 2y$
  - $f\left(\begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}\right) = f\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) + f\left(\begin{bmatrix} c \\ d \end{bmatrix}\right)$ for any $\begin{bmatrix} a \\ b \end{bmatrix}$, $\begin{bmatrix} c \\ d \end{bmatrix}$

  preserves the addition

- A function that sends every 2-tuple $\begin{bmatrix} x \\ y \end{bmatrix}$ to the 2-tuple $\begin{bmatrix} 2x + y \\ 3x \end{bmatrix}$ is a **linear function** from R^2 to R^2
  - We may write $g\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x + y \\ 3x \end{bmatrix}$
  - We may write $\begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x + y \\ 3x \end{bmatrix}$
  - $g\left(\begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}\right) = g\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) + g\left(\begin{bmatrix} c \\ d \end{bmatrix}\right)$ for any $\begin{bmatrix} a \\ b \end{bmatrix}$, $\begin{bmatrix} c \\ d \end{bmatrix}$

  preserves the addition

- Note that the following are **affine** functions (not linear):
  - $f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = x + 2y + 5$ from R^2 to R
  - $g\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x + y \\ 3x \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ from R^2 to R^2

mod n arithmetic

- mod 2 ----- Binary numbers = {0,1}
  - 0+0=1+1=0
  - 1+0=0+1=1
  - 0x0=0x1=1x0=0
  - **1x1=1**

mod 3 ------- Ternary numbers = {0,1,2}

- 0+0=1+2=2+1=0
- 1+0=0+1=2+2=1
- 1+1=0+2=2+0=2
- 0x0=0x1=1x0=0x2=2x0=0
- **1x1=2x2=1**
- 1x2=2x1=2

- mod 4 ---- Quaternary numbers = {0,1,2,3}

x	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	**0**	2
3	0	3	2	1

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

Examples and Exercise Problems

- Consider the binary numbers $\{0,1\}$. This is the integers mod 2. The addition/multiplication is done mod 2.
- Consider **all** the 2-tuples $\left\{\begin{bmatrix}0\\0\end{bmatrix},\begin{bmatrix}0\\1\end{bmatrix},\begin{bmatrix}1\\0\end{bmatrix},\begin{bmatrix}1\\1\end{bmatrix}\right\}$ over $\{0,1\}$.
- We let $A = B = \left\{\begin{bmatrix}0\\0\end{bmatrix},\begin{bmatrix}0\\1\end{bmatrix},\begin{bmatrix}1\\0\end{bmatrix},\begin{bmatrix}1\\1\end{bmatrix}\right\}$.
  - How many functions are there from A to B?

    ANSWER: $\mathbf{4^4 = 256}$

    Each member in A can be mapped into any one of 4 members in B.
  - How many linear functions are there from A to B?

    ANSWER: $\mathbf{2^4 = 16}$

    Any linear function from A to B is a 2 x 2 matrix over F_2

    and any 2 x 2 matrix over F_2 is a linear function from A to B.

    Therefore, the linear functions are of the form $\begin{bmatrix}a & b\\c & d\end{bmatrix}$ for $a, b, c, d \in F_2$.

    we will study this later !

Problems (Work on these problems and ask questions)

1. How many **1-1** functions are there from A to B?
2. How many **1-1 linear** functions are there from A to B? Describe each of them clearly.
3. How many functions are there from A **onto** B?
4. How many **linear** functions are there from A **onto** B? Describe each of them clearly.
5. Find all the functions which are linear, 1-1 and onto.
6. Find all the functions which are 1-1 and onto but NOT linear.

End of Introduction